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THESIS

A MODERN NAVAL COMBAT MODEL

by

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A Modern Naval Combat Model

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ABSTRACT

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I. INTRODUCTION

A. BACKGROUND

As war becomes more complex, the importance of combat models becomes increasingly relevant. But what is a model and what are its features?

A mathematical model is a mathematical construct which is designed to study a particular real-world system or phenomenon. A model can be a formula, equation, or system of equations describing how the underlying factors are interrelated [Ref. 1:p. 32]. In other words, "a model is a simplified representation of the entity it imitates or simulates." [Ref. 2:p. 1]

What is the general purpose of a combat model? There are two main purposes. First, a combat model can be considered as a tactical and decision-aid tool to help the decision maker. This is well summarized in the statement,

> A model is useful if a better decision can be made with the information that it adds. [Ref. 2:p. 17]

Experience from historical naval combat leads to the conclusion that a reliable and credible model can decisively help a commander make more sophisticated decisions in the combat arena. A rapid and intelligent decision can change drastically the outcome of a naval combat. It is not claimed,

however, that theory alone can win a battle. Leadership, morale, well-trained personnel, a vital and wise doctrine, and technological developments are chief among the factors that also count in determining the outcome. It is claimed that a good model greatly helps a leader make quicker, and usually wiser, decisions. As Burke[1] pointed out, the difference between a good leader and a bad one is about ten seconds [Ref. 3:p. 190].

The second purpose of a validated combat model is to aid in studying historical battles. Humans can learn from the mistakes of their predecessors, taking advantage of useful and valuable lessons. Col. T.N. Dupuy, U.S. Army Ret., has stated:

> History provides a base from which the anticipated effect of the new technology can be judged [Ref. 4:p. xxi].

Further:

> The value of military history is that, when analyzed objectively and scientifically, it permits us to project forward the trends of real past experience. This is the only way the relevant lessons of actual combat can be brought to bear on the important national defense issues of today. [Ref. 4:p. xxvi]

Using a model to analyze historical battles, analysts are able to answer some important questions. Among these are:

- In a particular situation, did the commander make the right decision?

[1]Arleigh Burke, a famous tactician, who defeated the Japanese at the battle of Cape St. George on November 25, 1943.

- If he had used the model, would he have persisted in his decisions or would he have changed his tactical plans?

An important factor of every combat model is the measure of effectiveness (MOE). What is chosen ultimately to measure? How do we assess or predict the outcome of an engagement? Do we consider attrition on both sides, probability of winning, accomplishment of the mission, or combat power for each side? Two other MOE options are: the total percentage loss of each force after the engagement, and the expected time until a certain percent of the enemy's combat power is destroyed. MOEs depend also on the level of command. That is, for the same mission, the chief of headquarters, the force commander, and the captain of a unit may be interested in measuring different outcomes. Therefore, who will use the information, and to what purpose, must be considered in deciding what the model output will be.

B. HUMAN FACTORS RELATED TO NAVAL COMBAT MODELS

Most of today's combat models do not take into account human performance under the specified conditions of the model. In existing models, the representation of people--the real combatants--is assumed to be deterministic. However, as experience has taught, human performance and action greatly influence the outcome of an engagement in many ways and they generally do not vary in a deterministic manner. Combat models

will reflect real-world conditions only if actual data about human performance is included.

The difficulty of incorporating human performance data in combat models stems from the intangibility of the human variables that are involved. That is, the human variables are very difficult, if not impossible, to quantify with any degree of confidence because they are essentially qualitative in nature [Ref. 5:p. 34].

Recognizing that numerous human factors issues affect naval combat either directly or indirectly, T.N. Dupuy, in association with colleagues from the Historical Evaluation and Research Organization, has developed a methodology to include several human-related variables. Called the Quantified Judgment Model, or simply QJM, Dupuy's method introduces a factor Q, for troop quality. This factor includes the human factors of leadership, morale, and training, as well as logistical capability, intelligence, initiative, command and control, communications, momentum, time, and space [Ref. 4:p. 106]. Because this Q factor is not directly measurable, Dupuy uses the term "combat effectiveness" in place of troop quality. In his usage <u>combat effectiveness</u> reconciles the difference between results based on theoretical combat power and actual battle results. In the narrow frame of this thesis we will consider, and briefly describe, only the three major

human factors included in Dupuy's QJM: leadership, morale, and training.

1. **Leadership**

In his book Fleet Tactics [Ref. 3], Capt. Wayne P. Hughes, Jr., emphasizes the importance of a good leader in a naval battle. If leadership is an important human factor in land combat, then, because of the peculiarity of a warship, it is of essence in naval warfare. On board a ship there is no alternative: everybody goes where the leader goes and everybody shares with the leader a common fate. Capt. Hughes talks about the "mystique" or "charisma" of leadership [Ref. 3:p. 26]. Further, he states:

> ...Shall we say, Know your forces, know your enemy and know yourself? This is all splendid advice, but I would argue that nothing takes precedence over the peacetime commander's job of finding combat leaders. Let him do his best to find them, send them to sea, and keep them at sea longer than the U.S. Navy does now. Let the first aim of every seagoing commander be to find two officers better than himself and help in every way to prepare them for war. That done, everything else will follow. [Ref. 3:p. 195]

We would add here that what is likely to follow is better command and control, more effective scouting, and concentration of power. Hughes' maxim, "Attack effectively first," will then be applied, and potential destructive losses can be converted into glorious victories.

It should be noted that it would be a fatal mistake of a commander in preparing his tactical plans to overestimate his capabilities and underestimate the abilities of the leader of

his opponent force. Before the battle a commander should consider the leader of the enemy to be at least as good as himself.

2. Morale

Morale may depend heavily on having a good leader. We believe also that morale and training go together. It seems reasonable to assume that, with well-trained personnel under a good leader, it would be unlikely to see panic in a difficult situation. We also believe that one factor that plays an important part in personnel morale is a certain "quality of character" that depends neither on the education nor on the training of the personnel. One word to describe that quality is "ethos."

According to Peter Watson, other factors affecting morale can be summarized as follows:

- <u>The results of the first encounter</u>. If the first battle has been fought and won, this successful encounter will help morale rise.
- <u>The emotional support provided by informal leaders</u> (those who "take charge," whether or not they have formal authority).
- <u>The number of casualties incurred</u>. Reducing physical casualties helps greatly in maintaining high morale.
- <u>The cohesiveness of the groups</u>. Morale is much higher if personnel are trained in small groups and kept together all the time. These "teams" have better *esprit de corps*. [Ref. 6:pp. 231-232]

Is it possible to quantify morale? As with all intangible factors, this question cannot be answered with certainty. However, Dupuy proposes a set of numerical values for various levels of morale. These are shown in Table I below [Ref. 5:p. 231].

TABLE I. DUPUY'S QUANTIFICATION OF MORALE LEVELS

Level of Morale	Assigned Value
Excellent morale	1.0
Good morale	0.9
Fair morale	0.8
Poor morale	0.7
Panic	0.2

3. Training and Experience

If a substantial difference exists in the level of training and experience between the two opposing sides, the outcome of the battle may be influenced greatly. For this reason it is essential that during peacetime everyone involved be as professionally trained in his domain as possible. Aboard a warship everyone acting individually, as well as in a group, during combat should know exactly what to do and when to do it.

History indicates that chance or luck sometimes greatly influences the outcome of naval combat. Sometimes a good leader seems "lucky" when he exploits all his opportunities perfectly. Yet, what may be considered pure chance by an inexperienced analyst may actually be a mixture of experience and boldness. On the other hand, luck always helps even bold and brave people. Though boldness and bravery are important, it would be a mistake to disregard the influence of chance or luck. It should be noted here that, in some countries, one quality used to grade Naval officers is how lucky they appear to be.

C. THESIS GOAL AND SCOPE

The goal of this thesis is to develop a reasonable modern naval combat model which, after it has been appropriately validated, could be used for two primary purposes. These are:

- For better understanding of historical naval battles.
- As a tactical or decision aid "tool" that a commander may use to assess his tactical plans.

For the latter purpose, it is assumed that the tactical commander uses the model for a reasonable length of time, and that he is able to estimate the composition and state of the enemy force (using scouting efficiently). Moreover, it is assumed he knows the exact situation (state) of the combat force assigned to him.

In order to develop a model, information is presented as follows. In Chapter II an extensive naval combat theory is described and appropriate terminology introduced. This is intended to help the reader better understand the concepts that follow.

In Chapter III two different model approaches are described. The reason for this is to introduce the reader to the world of combat models, and to give a sense of how such models work and their weak and strong points. The first of the two models to be presented is Dupuy's Quantified Judgment Model (QJM), incorporating modifications proposed by Capt. Joseph Ciano, U.S. Marine Corps [Ref. 7]. A brief description and explanation of the model are provided, including a presentation of its main equations. The weaknesses of the model are also discussed.

The second modeling approach is based on Capt. Hughes' concepts about naval warfare. This approach was developed by Lt. Thomas Reagan Beall, U.S. Navy [Ref. 8]. The model is described briefly, including how it functions and its weaknesses. The two modeling approaches are then compared.

Chapter IV is devoted to the development of a modern naval combat model that includes various human-related factors. This combat model is based on Hughes' model (as developed in Beall's thesis [Ref. 8]). We also attempt to correct some of the weaknesses of the Dupuy and Beall models. First, the

problem is identified and the assumptions are stated. An indirect method of quantifying the intangible human factors of leadership, morale, and training follows in order to integrate them into the model.

Only pulse weapons (specifically missiles) are considered in developing the model. Pulse weapons are considered those weapons which deliver instantaneously an enormous amount of combat power against a target. Missiles, torpedoes, and bombs are examples of pulse weapons [Ref. 8:p. 6]. This limitation is based on the belief that today's naval warfare can be considered salvo warfare; that is, combat based on modern naval surface missiles. The immense technological development since World War II drives us to believe that the idea of gradual attrition of one force by the other is obsolete. Also included in Chapter IV are several numerical examples, in an attempt to shed light on how the developed model works.

Conclusions concerning the credibility of the model are summarized in Chapter V. Recommendations for further research and validation of the model are also included there.

Appendix A contains computer code (Fortran 77) illustrating how the model developed in Chapter IV works under specific assumptions. Also included there are the computer outputs of the examples used in Chapter IV.

II. NAVAL COMBAT THEORY

A. INTRODUCTION

This chapter is devoted to a general discussion of combat theory and terminology in order to help the reader understand the models that are addressed in the next several chapters. Discussion is limited to the most basic ideas necessary for familiarity with naval combat and for understanding a naval combat model. The goal of this chapter is to lay the foundation for the developments to follow and to establish a framework of terminology.

The basic concepts analyzed in this chapter are drawn from J.T. Swanson and J.H. Gibson (Reference 9, pages 8 to 29), and Capt. Hughes (Reference 3, and Reference 10, pages 2 to 16). Hughes' extensive knowledge of the subject guarantees that the concepts and ideas developed below are the product of many years experience.

B. COMBAT THEORY

1. Command and Control

The importance of leadership has already been discussed, but what are the duties and qualities of a leader? They are known to be associated with the phenomena of command and control, but what is meant by "command and control?"

If each term is examined separately, command and control both represent military functions. <u>Command</u> is the function that organizes, motivates, and decides what is needed from forces. <u>Control</u> executes the command decisions. In other words, control is the function that transforms the perceived need into action [Ref. 3:p. 147].

<u>Command and control</u> (C2) is a process involving a person who is responsible for making decisions and causing those decisions to be executed. These two ideas, decision making and execution of the decisions, are the key elements of the command and control process.

From a military point of view the decisions made by a commander concern military organizations and operations. The main purpose of a military organization's existence is to carry out activities related to armed conflicts. Thus, every decision concerning military organizations should be based on one key concept: enhance the fighting ability of the military organization in combat. The essence of a command and control system is to analyze, evaluate, and find ultimately what action should be carried out to direct the organization in accomplishing its mission. Communication is included in the command and control process. Thus it is not necessary to define command, control, and communication (C3) as a separate concept here.

2. Combat and Forces

Military organizations are made up of forces. Personnel and equipment are components of a force. We need to distinguish the idea of a military force as personnel and equipment from the concept of force in its physical sense of "compulsion exerted in a battle." We define force as the warships and aircraft that create the compulsion exerted on the battlefield. We will call this compulsion "combat power." Combat is then defined as an interaction of force-on-force activities. The tools used to carry out the battlefield activities are the military forces.

An element of a force is a unit (a ship, for example) of which the force is composed. An element is characterized by specific features, such as the number of personnel, type of weapons, ammunition availability, its geographic location, and so forth. In naval operations, an element is typically a ship. Several elements of force, taken together with a common task, constitute a group. For instance, the common task could be delivering firepower to the enemy, or disrupting the combat functions of the enemy. How are the various terms that are defined above related to one another? The following statement gives some insight.

> In combat each element of a force will perform actions based on the function assigned to the element (by command), the current state of the element (capability of the element at a given time) and the attributes of the element. For example, an AAW unit will perform actions against enemy aircraft, but it is not expected to take

effective action against enemy infantry or armor. [Ref. 9:p. 12]

Combat activity is defined as the change in the state of both sides caused by an element-action-element exchange. The collection of these activities causes a change in the total force of both sides.

3. Combat Potential

The combat potential of an element (ship) or force is represented by the weapons and ammunition it carries. Combat potential includes the ammunition stored in the magazines as well as ammunition loaded in guns, launchers, and so forth. It can be considered as the overall capacity of a particular force to carry out combat activities successfully against an enemy.

The combat potential characterizes the current state of a given force and is categorized into two types: designed combat potential and available combat potential. The designed combat potential is the (designed) capacity of a given force to achieve its full potential given a known enemy and given that all the other factors affecting the combat are optimal (optimal scouting, optimal leadership and training, and optimal functioning of equipment). This combat potential can be considered as a theoretical entity in the sense that it represents the best a given force can do (all other factors considered as optimal).

The available combat potential of a force (for a specified combat mission) is its current or actual capacity to achieve successful results, given existing levels of scouting, leadership, and training. Clearly the available combat potential of a given force in a given battle is less than its designed combat potential (usually some fraction between 0.0 and 1.0 of the designed combat potential) [Ref. 9:p. 14].

4. Combat Power

A very important aspect of combat power is that it is specified only for a certain mission in a particular environment against a particular enemy. This characteristic is due to the fact that combat power is a real phenomenon in a battle: it is the actual capability of the force to achieve results in a combat. When the forces are activated by a commander in the command and control process, combat power is produced. Combat power is considered to be the rate of force projection, that is, shells, torpedoes, bombs, or missiles fired against an enemy by the forces within a command. The following few sentences describe in an effective and concise way the concept of combat power and its relationship to the command and control process, force, and combat potential.

> Combat power is generated against an enemy as a result of forces carrying out combat actions against the enemy based upon a commander's activation of his forces, utilizing a command and control process. The combat power is generated from available combat potential of the forces involved, but does not necessarily consume the potential of the forces in the way that energy is consumed from a battery during its use. [Ref. 9:p. 14]

Combat power can thus be seen as a function of the number of a force's elements on the one hand, and the type of forces and rate of their activities on the other. If P denotes combat power, **m** the number of elements in a force, and **u** the rate of the force's activities, then the fundamental equation of combat power is given by

$$P = F(m,u)$$

where F is called the command function. [Ref. 9:p. 15]

Another important differentiation in combat power is the distinction between theoretical and effective combat power. Theoretical combat power can be considered roughly as the quantity of ammunition the specific unit (platform) can fire per time unit or per salvo (for pulse weapons). On the other hand, effective combat power or simply combat effectiveness is the combat power that results in attrition to the enemy. It can be represented as the number of hits per time unit[2].

5. **Combat Effectiveness**

Combat effectiveness can be thought of as the attrition of the enemy caused by delivered combat power. Combat effectiveness is measured by combat results during a

[2]According to the theory, other effects are sometimes important. For example, suppression of enemy fire and movement, or interdiction of reinforcements and supplies. These effects will not be treated in this thesis.

battle, and <u>combat outcome</u> is the cumulative attrition of the enemy achieved by combat power[3].

Another distinction is now apparent. The results of combat functions performed by Side A (that is, the effect of combat power exerted by it) depend on Side B's defensive actions. Side B is applying countermeasures (such as jamming, evasion, and so forth) to lessen the effectiveness of Side A's combat power. Side B is likewise applying offensive activities that generate its combat power against Side A while side A attempts to attenuate these activities.

Thus both sides simultaneously apply offensive activities, thereby creating combat power against the opponent, as well as defensive activities to lessen the effect of the opponent's combat power. It is now clear why combat is defined as an interaction of force-on-force activities.

The above definitions and interpretations are summarized in the following paragraph.

> ...command is the all encompassing function which generates the designed and available combat potential. Through the subfunctions of organizing, motivating, deciding and executing, a commander brings his forces from some untrained or otherwise unready condition to a point where the available combat potential of the forces is as near as possible to its designed combat potential. The readiness of the forces prior to executing an operation is the responsibility of commanders at many echelons and is accomplished through the function of command. [Ref. 9:p. 18]

[3]The theory does not require that results be measured as attrition, but that there is some suitable measure for naval combat.

6. Scouting and Antiscouting

Scouting and antiscouting have played an important role in naval history, from earliest times through the present. These processes can definitely decide the outcome of naval combat and sometimes give the victory to the "inferior" side. <u>Scouting</u> is the gathering of useful combat information. Using this definition, acts of search, detection, tracking, targeting, surveillance, reconnaissance, and cryptanalysis are considered as scouting [Ref. 3:p. 146]. Scouting is achieved only when the information is actually delivered to the tactical commander for considered action.

<u>Antiscouting</u> includes all actions taken to lessen, destroy, or diminish the enemy's scouting effectiveness. To shoot down a reconnaissance aircraft, jam its radar, or use any other means to reduce the ability of an enemy to gather information is considered antiscouting.

7. Other Important Definitions

<u>Command and control countermeasures</u> (C2CM) are actions carried out by Side A to reduce the effectiveness of Side B's command and control process. The result of such countermeasures can be that Side B cannot effectively activate its forces, thereby decreasing its combat power. For example, in the effort of Side B to establish communications between its units (C2 process), Side A answers by jamming the

frequencies used by B, therefore trying to prevent Side B from effectively activating its forces.

Generally each combat process on one side has a countermeasure on the other side. The combat processes, together with their countermeasures, are summarized in Table II. [Ref. 10:p. 3]

TABLE II. RELATIONSHIPS BETWEEN COMBAT PROCESSES AND COUNTERMEASURES

Processes	Countermeasures
Attrition (destruction or damage)	Protection
Suppression	Covering
Scouting (information acquisition)	Screening[4]
Supply (or support)	Interdiction
Maneuver (or motion)	Fixing (including disruption)
Command and control (including communication)	Counter-C2 (including deception)

[4]In naval terminology, escorts forming a screen perform both the screening and protection processes.

Firepower is the capacity (potential) to destroy the enemy's ability to deliver combat power [Ref. 3:p. 146]. An element (a ship, for example) has suffered a firepower kill if its combat power diminishes to zero, so it cannot contribute combat power to its force.

Staying power is the capacity of a specific platform to absorb damage and continue fighting with some measurable effectiveness [Ref. 3:p. 289]. It is a measure of how many hits the platform can absorb before losing its combat effectiveness. Staying power is often the measure of how much damage the platform can absorb before becoming useless (but not necessarily sunk) in the naval battle under consideration.

8. Continuous Versus Pulse Weapons

Continuous weapons are those weapons causing a continuous rate of attrition to the enemy (for example, main battery guns of a platform). On the other hand, weapons that are able to deliver instantaneous and great doses of substantial combat power in pulses over long distances against a target are called pulse weapons (torpedoes and missiles, for example). We then distinguish the following terms:

- Continuous versus pulse theoretical combat power.
- Continuous versus pulse weapon effectiveness.
- Continuous versus pulse effective combat power.

9. Uncertainty in Naval Combat

"Uncertainty is inherent in combat" [Ref. 11: p.6-1]. This statement is included as one of the six basic axioms that have been adopted by The Military Conflict Institute (TMCI) in order to understand and then model a combat situation. Uncertainty is defined as "a state of doubt about the combat situation, including the outcome" [Ref. 11:p. 6-2]. Uncertainty is often introduced by scouting inadequacy which results in incomplete information about the enemy (personnel, equipment, intentions, and so forth). Other sources of uncertainty include the enemy's efforts for deception, and doubt about the exact state of our own force at the time of combat. The actual combat potential that will be activated by the commander to generate combat power cannot be predicted in advance (this is the difference between designed and actual combat potential).

Even if everything concerning the enemy is known with certainty, uncertainty about the outcome of the combat (although reduced) will remain. This uncertainty is due to the human factors involved. Since humans operate the military equipment and make the decisions, it can be assumed that the outcome of a battle will not be deterministic, regardless of how good the technological development happens to be. A combat leader certainly can make predictions using combat models that are based on theoretical combat power or on the designed

combat potential of both sides, and so forth. However, to what extent the output of a model can be trusted is another question. How great must his model-predicted advantage be in comparison with the enemy in order to be certain (for all practical purposes) that his predicted victory will occur? The following example illustrates an extreme case that is deterministic. Side A is composed of one aircraft carrier and eight frigates. Side B is composed of two destroyers only. In such a case, the advantage of Side A is so great that, in this hypothetical engagement, the victory of Side A is deterministic. In other circumstances, when the advantage of the one side is not so relevant, scouting effectiveness, surprise, and other human factors may very well change the situation, giving the advantage to the hypothetically inferior (in terms of combat potential) side.

C. LANCHESTER'S MODEL[5]

Lanchester's model was one of the first combat models developed and is still considered today a very important model. Thus it is useful to describe it in this discussion of combat theory.

[5]The concepts developed in this section were taken primarily from lectures given by Prof. Samuel Parry in 1989 at the Naval Postgraduate School, Monterey, California. Also used are Reference 12, pages 6 to 10, and Reference 1, pages 369 to 375.

Let X and Y be two forces, let x(t) and y(t) be the sizes of these forces at time t, and let a and b be the attrition rate coefficients for the X and Y forces, respectively.

1. Square Law

Two equations, commonly referred to as the <u>square law</u>, are used to represent a situation where attrition to each side is proportional to the number of units remaining on the other. These two equations are presented in Figure 1.

$$\frac{dx(t)}{dt} = -ay(t)$$
$$\frac{dy(t)}{dt} = -bx(t) \qquad (1)$$

Figure 1. Lanchester Model Square Law

This law can also be considered as a concentration or "aimed fire" law (a concept of modern warfare). This is so because each shot from a unit of the X forces has a certain probability of eliminating the unit of Y forces at which it is aimed, with a probability of zero for eliminating any other unit.

Using the square law, the attrition coefficient a represents the quantity of casualties of X per firer of Y per

minute. If x_0 and y_0 are the initial sizes of the X and Y forces at the beginning of the engagement, and x_f and y_f are the final numbers of survivors for each side, then the state equation obtained by solving the differential equations of the square law is

$$b(x_0^2 - x_f^2) = a(y_0^2 - y_f^2). \tag{2}$$

The state equation can be used to determine the value y_f if x_f is zero, for instance. Or it can be used to determine another value of y_f, called the "break point", when say, $x_f = 0.2x_0$). It is also possible to determine the necessary and sufficient condition for a force to win. If the battle is continued to its finish (so either $x_f = 0$ or $y_f = 0$) then

$$X \text{ wins if } \frac{x_0}{y_0} > \sqrt{\frac{a}{b}},$$

and

$$Y \text{ wins if } \frac{x_0}{y_0} < \sqrt{\frac{a}{b}}.$$

To determine the size of each force at time t, it is necessary to convert the system (1) to a single second-order differential equation involving x and y. To do this we differentiate the first of the two equations in system (1) and

substitute the second, obtaining the following second-order differential equation:

$$\frac{d^2x}{dt^2} - bax = 0. \tag{3}$$

Solving this equation yields the size of the X force at time t:

$$x(t) = \frac{1}{2}\left(x_0 - \sqrt{\frac{a}{b}}y_0\right)e^{\sqrt{ab}t} + \frac{1}{2}\left(x_0 - \sqrt{\frac{a}{b}}y_0\right)e^{-\sqrt{ab}t} \tag{4}$$

where x(t) is the force level of X at time t.

To determine how long the battle will last, assume that Y wins. Then x(t) is set equal to zero in Equation (4) and solving for t gives the result

$$t = \frac{1}{2\sqrt{ab}}\ln\left(\frac{(1 + \frac{x_0}{y_0}\sqrt{\frac{b}{a}})}{(1 - \frac{x_0}{y_0}\sqrt{\frac{b}{a}})}\right). \tag{5}$$

2. Linear Law

Another differential equation model is referred to as the linear law. This law models area or unaimed fire[6]. The equations modeling the linear law are given in Figure 2.

$$\frac{dx(t)}{dt} = -ax(t)y(t)$$
$$\frac{dy(t)}{dt} = -bx(t)y(t) \qquad (6)$$

Figure 2. Lanchester Model Linear Law

Each shot that an X unit fires will eliminate all Y units within some lethal area. All Y units are considered to be uniformly distributed over the area that the Y force occupies.

A linear law battle can be thought of as lasting for an infinite length of time. With the linear law the derivative dx/dy is the constant b/a, where the attrition coefficient a represents the quantity of casualties of X per firer of Y per

[6]The linear law, in a slightly different mathematical form, can also be used to model a series of duels. This form was said by Lanchester to describe ancient warfare, and it has the linear law's state equation.

target of X per minute (with a similar interpretation for the attrition coefficient b). The state equation is given by

$$b(x_0 - x_f) = a(y_0 - y_f). \tag{7}$$

The X force level as function of time is given by

$$x(t) = \begin{cases} x_0\left(\dfrac{bx_0 - ay_0}{bx_0 - ay_0 e^{-(bx_0 - ay_0)t}}\right), & if\ \ bx_0 \neq ay_0 \\ \\ \dfrac{x_0}{1 + bx_0 t}\ , & if\ \ bx_0 = ay_0\ . \end{cases} \tag{8}$$

We may also have a situation in which the X force is in the open field, and the Y force ambushes X (so that Y is considered "hidden"). The Y force will conduct aimed fire against X, and X will be attrited according to the square law. At the same time X will conduct area fire against Y and Y will be attrited according to the linear law. This mixed combat is modeled by the differential equations in Figure 3.

$$\frac{dx(t)}{dt} = -ay(t)$$
$$\frac{dy(t)}{dt} = -bx(t)y(t) \tag{9}$$

Figure 3. Lanchester Model Mixed Law

In this situation the state equation is given by

$$\frac{b}{2}(x_0^2 - x_f^2) = a(y_0 - y_f). \tag{10}$$

This presentation of Lanchester's models here was for the purpose of making it clear that the Lanchester models are inadequate descriptions for modern naval combat. These models do not take into account scouting and antiscouting effects, staying power of a unit, or the effect when a force consists of different kinds of ships. Most important of all, the effect of pulse weapons (instantaneous delivery of substantial combat power) is not taken into account by the Lanchester models.

III. SUMMARIES OF TWO COMBAT MODELS

A. QUANTIFIED JUDGMENT MODEL (QJM)

1. Model Development

The Quantified Judgment Model (QJM) has been referred to in the previous chapters. It is a methodology developed by Col. T.N. Dupuy, U.S. Army, Ret., to assess historic land combats. In order to account for weapon evolution, Dupuy adapted Clausewitz's concepts for today's reality. Dupuy begins his development by formulating an equation to represent what Clausewitz had in mind for determining a battle's outcome. All Clausewitz's theory about combat, which Dupuy calls the "Law of Numbers," is summarized in his single equation. [Ref. 4:p. 30] This equation is

$$P = N \times V \times Q \tag{1}$$

where: P = Combat power

N = Numbers of troops

V = Variable circumstances affecting a force in battle

Q = Quality of force.

Dupuy also derived an equation to represent Clausewitz's concept of a battle outcome [Ref. 4:p. 30]. The battle outcome is defined by the following ratio of combat power of the Red force to combat power of the Blue force:

$$Outcome = \frac{N_r \times V_r \times Q_r}{N_b \times V_b \times Q_b} \qquad (2)$$

where: r = Red force identifier

b = Blue force identifier.

In order to develop his model, Dupuy made several substitutions in Clausewitz's law of numbers. First, Dupuy substituted force strength (S) for numbers of troops (N). Then he introduced the operational factor V_f to replace V and created his combat power equation as follows: [Ref. 4:p. 87]

$$P = S \times V_f \times Q \qquad (3)$$

where: S = Force strength

V_f = Operational and environmental factor

Q = Troop quality factor.

Force strength S is a function of number of weapons, lethality of weapons, and weapon effects (terrain, weather,

season, and air superiority) [Ref. 7:p. 45]. Operational and environmental factors V_f "represent the effect of the circumstances of the combat on the force" [Ref. 4:p. 81]. Troop quality Q represents and incorporates several human factors affecting a battle (leadership, morale, training, and experience).

To compare power between forces a Combat Power Ratio between the Red and Blue forces is defined by Dupuy as follows:

$$\text{Combat Power Ratio} = \frac{P_b}{P_r}. \qquad (4)$$

After the derivation of the combat power Equation (3), an important question comes into play: how do you assign a numerical value to Q? In order to quantify Q, Dupuy took another step. He substituted relative combat effectiveness for troop quality (Q). To define combat effectiveness for QJM, we need to define theoretical combat power and the actual battle results equation.

Theoretical combat power (P') of a force can be thought of as the designed combat power for this force. It is a function of the force strength as well as operational and environmental factors. As expected, it is expressed much the same as combat power as defined above, after the troop quality

factor Q is omitted (or set "ideally" to the value of 1). Formally, theoretical combat power is given by

$$P' = S \times V_f. \qquad (5)$$

Similar to Equation (4), the theoretical combat power ratio between the Blue and Red forces is defined by the equation

$$\textit{Theoretical Combat Power Ratio} = \frac{P_b'}{P_r'}. \qquad (6)$$

The theoretical combat power ratio represents a prediction of the outcome of a hypothetical engagement between Blue and Red forces when there is no difference in fighting quality between them.

The term actual battle results is used by Dupuy to represent the actual outcome of a historical battle. It is a function of three factors: mission accomplishment, ability to hold or gain a ground, and force effectiveness when casualties have occurred. These three factors are defined as follows:

- Mission factor (MF): an "expert" judgment of the extent to which a force accomplished its assigned or perceived mission.
- Spatial effectiveness (Esp): a value representing the extent to which a force was able to gain or hold ground.
- Casualty effectiveness (Ecas): a value representing the efficiency of the force in terms of casualties, taking into consideration the strengths of the two sides and the casualties incurred by both sides. [Ref. 4:p. 88]

These three factors are summed to give the actual battle results. The actual battle results equation is

$$R = MF + Esp + Ecas. \tag{7}$$

The mission accomplishment factor is purely subjective and values are assigned to it by Dupuy on a scale from 1 - 10, as presented in Table III. [Ref. 5:p. 231]

TABLE III. DUPUY'S MISSION FACTORS

Mission Description	Range	Normal
Complete accomplishment of the mission	7-10	8
Substantial, relatively satisfactory accomplishment	5-7	6
Partial, less than satisfactory accomplishment	3-5	4
Little achievement of the mission	1-3	2

Spatial effectiveness (Esp), which is the ability to hold or gain ground, is a function of the relative strengths of the two forces, the relative depths of the areas occupied by each, the average daily distance of advance or withdrawal, and the military posture of forces (attack, defense, and so forth)[7] [Ref. 7:p. 48]. It is not necessary for purposes of this thesis to define or analyze the equations representing Esp.

Casualty effectiveness (Ecas) is a function of the relative average daily number of casualties, the relative force strength, the military posture of the forces, the total

[7]Spatial effectiveness is defined mathematically in Reference 5, page 48. A modified equation that makes more sense from a military standpoint is given in Reference 7, page 53.

number of personnel in the force, and the vulnerability of the force[8].

After calculating the results for both the Red and the Blue forces, the actual outcome of the battle is given by the Actual Battle Results Ratio as follows:

$$Actual\ Battle\ Results\ Ratio = \frac{R_b}{R_r}. \tag{8}$$

It is not reasonable to assume that the theoretical outcome in Equation (6) would be the same as the actual outcome in Equation (8) because human factors almost always affect the battle (as well as the factor of chance or luck). So it is plausible to say that the difference between the actual battle results ratio (actual outcome) and the theoretical combat power ratio (theoretical outcome) is due chiefly to human factors.

From the previous definitions and interpretations, we now define combat effectiveness (CEV) as the ratio between actual and theoretical outcome. In mathematical terms, combat effectiveness for the Red force is defined as follows: [Ref. 4:p. 89]

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[8]A mathematical formula of Ecas can be found in Reference 5, page 49.

$$CEV_r = \frac{\frac{R_r}{R_b}}{\frac{P'_r}{P'_b}}. \qquad (9)$$

Dupuy defines the combat effectiveness of the Blue force (CEV_b) as the reciprocal of the combat effectiveness for the Red force (CEV_r). It seems reasonable that the combat effectiveness for the Red force is equivalent to its troop quality factor (Q). Therefore, Dupuy redefines combat power for the Red force as

$$P_r = S_r \times V_{f_r} \times CEV_r = P'_r \times CEV_r. \qquad (10)$$

Equivalently, substituting CEV_r from Equation (9), we have

$$P_r = \frac{R_r}{R_b} \times P'_b. \qquad (11)$$

Finally, in order to use the model to analyze historical battles, a new combat power ratio is defined. The combat power ratio defined in QJM [Ref. 7:p 33] was found to

be inconsistent by Ciano during his study of the model [Ref. 7]. Ciano subsequently proposed a <u>relative combat power equation</u> which is presented in Figure 4. This equation is the final product of QJM and may be used to evaluate the outcome of an historical combat.

$$\text{Relative Combat Power (Blue force)} = \frac{S_b \times V_{f_b} \times \sqrt{CEV_b}}{S_r \times V_{f_r}}$$

$$\text{Relative Combat Power (Red force)} = \frac{S_r \times V_{f_r} \times \sqrt{CEV_r}}{S_b \times V_{f_b}}$$

Figure 4. Ciano's Relative Combat Power Equation for the QJM

The force that possesses a relative combat power greater than 1 is considered the superior force and would win.

2. Strengths and Weaknesses of Dupuy's Model

The QJM is a very useful model. It draws considerable information from historical battles and yields important conclusions for the future. It includes Clausewitz's theory and is based on Dupuy's many years of experience. It was developed by Dupuy and his associates over a considerable

period of time and was validated against historical real-world combat data. Although it has already been used to study historical battles, its basic equation for relative combat power (the one that was used to determine the outcome of a combat) was found to be inconsistent by Ciano, as reported in his thesis. The modification, suggested by Ciano in Figure 4, is very reasonable and seems to work well.

A point of weakness in the QJM is the definition of the mission accomplishment factor; it is a completely subjective measure. Because of the form of the actual battle results Equation (7), mission accomplishment often dominates the more objective factors (losses and territory exchanged). To show one difficulty, note that losing a battle is sometimes considered a successful mission accomplishment by a level of command (if the enemy was delayed appropriately, or if the enemy suffered greater losses).

Another weakness in QJM is that force strength is based on historical data to obtain comparative values. Also, the model has so far proven most useful to analyze historical data. Though Dupuy claims great power for it, other Army analysts believe its value for predicting future outcome in a battle is uncertain. Moreover, the model does not account for the dynamics of combat such as tactics on the battlefield, the effect of maneuver and suppression, and the problem of

distribution of force over a battlefield to obtain the best combat power. [Ref. 9:p. 96]

Finally, Dupuy's model is designed for ground combat and cannot be easily adapted to naval combat without extensive revision, including research into historical naval battles.

B. A NAVAL COMBAT MODEL

The model to be discussed next is a pure naval combat model. As stated earlier, it was developed by Lt. Thomas Beall in his thesis and is based on Hughes' naval warfare concepts [Ref. 8:pp. 5-31]. (The reader who is interested in more details should refer to that thesis.) Although the model deals with both continuous and pulse weapons, we will concentrate here on the continuous weapon case for two reasons:

- We are going to deal with pulse weapons in the next chapter, and
- Missiles are today's most important naval weapons but they are not included in Beall's work.

Beall's model serves two purposes. First, it may be used to study historical naval battles and compare their actual outcomes with the model predictions. Second, the model can be used as a tactical planning tool for a decision maker.

1. Definitions

The following definitions (together with the combat theory developed in Chapter II) are needed to understand Beall's model:

- 1000-pound bomb equivalent (TPBE): a measure of destruction. It is equal to the explosive power of 660 pounds of TNT (equivalent to the explosive power of a 1000-pound bomb in the Second World War). The explosive power of all weapons is expressed in multiples of TPBEs.
- Theoretical combat power (FC): the number of TPBEs per minute which a platform's main battery guns can fire.
- Weapon effectiveness (PC): the probability that a shell fired from a group's main battery gun will hit the aimed target.
- Effective combat power (EFC): the number of TPBEs that hit their targets per minute.
- Staying power (SP) of a platform: the number of TPBEs necessary to inflict a firepower kill on that platform.
- Indices are defined by Beall as follows:

  i = Weapon
  j = Platform
  k = Group
  l = Blue force
  l′ = Red force.

The function of the model is summarized in the following paragraph:

> Naval combat is modeled as a force-on-force attrition process. Component groupings of each force are portrayed as aggregations of the SP and FC values of their individual platforms. Attrition is computed in discrete time steps and is represented by the simultaneous degradation of each force's aggregate SP and FC over time. [Ref. 8:p. 7]

2. Characteristic Values for a Platform

The staying power, measured in TPBE hits, of platform j in group k in force l, based on historical data, was derived in Appendix A of Reference 8 as the expression

$$SP_{jkl} = 0.070 \times (full\ load\ displacement)^{1/3} \qquad (1)$$

where full load displacement is a characteristic of a ship[9].

The theoretical combat power of main battery gun i in platform j in group k in force l (that is, the number of TPBEs fired from the specific gun i per minute) is computed according to the formula

$$FC_{ijkl} = \frac{weight}{660 lbs} \times wtg \qquad (2)$$

where: weight = Explosive weight that the main battery gun fires per minute (in pounds of TNT)

wtg = 2.5, an empirically derived multiplier to account for the greater kinetic energy of the shell relative to a bomb at the impact point.

[9]We use the same notation as that adopted by Beall in his thesis.

The theoretical combat power of a platform j in group k in force l is given by summing the FC of each main battery gun in the platform

$$FC_{jkl} = \sum_{i \in j} FC_{ijkl}. \tag{3}$$

In the summation of Equation (3), platform j is fixed in the group k of the Blue force l. Thus the summation is taking place over all weapons i in that platform.

The values of the aggregate staying power (SP) and theoretical combat power (FC) of a group k in force l, fired as a single unit, are given by

$$SP_{kl} = \sum_{j \in k} SP_{jkl} \quad \forall k, \ \forall l, \tag{4}$$

and

$$FC_{kl} = \sum_{j \in k} FC_{jkl} \quad \forall k, \ \forall l. \tag{5}$$

The symbol $j \in k$ refers to platform j assigned to group k. In the summations k is fixed and j varies over all platforms in group k.

Finally, the effective combat power (EFC) of group k in force l is computed according to the equation

$$EFC_{kl} = FC_{kl} \times PC_{kl}. \tag{6}$$

3. **Model Description**

As mentioned above, the aggregate values of staying power (SP) and theoretical combat power (FC) of all the groups are recomputed in each discrete time step (say 1.0 minute). The terms $SP_{kl}(t)$ and $FC_{kl}(t)$ are the aggregates of SP and FC of group k in force l at time step t.

If l′ represents the attacking force, the aggregate value of staying power (SP) of the groups under attack is defined as follows:

$$TS(t) = \sum_{k \text{ being attacked by } l'} SP_{kl}(t-1) \tag{7}$$

where: $SP_{kl}(t-1)$ = Staying power of group k belonging to force l at the end of the (t-1) time step.

The aggregate effective combat power (AEFC) of the attacking groups is computed as follows:

$$AEFC(t) = \sum_{k \text{ firing from } l'} FC_{kl'}(t-1) \times PC_{kl'} \tag{8}$$

where: $FC_{kl'}(t-1)$ = Theoretical combat power of group k belonging to force l' at the end of the (t-1) time step.

The ratio of AEFC to TS is defined as the defender's continuous fire loss percentage (LC). Thus,

$$LC = \frac{AEFC}{TS}. \tag{9}$$

Finally, if the continuous fire loss percentage (LC) is considered to be applied to the staying power (SP) and theoretical combat power (FC), these values can be computed for each iterative time step as follows:

$$SP_{kl}(t) = \begin{cases} SP_{kl}(t-1) \times (1-LC) & \forall k \text{ under attack} \quad (10) \\ SP_{kl}(t-1) & \text{otherwise.} \quad (11) \end{cases}$$

$$FC_{kl}(t) = \begin{cases} FC_{kl}(t-1) \times (1-LC) & \forall k \text{ under attack} \quad (12) \\ FC_{kl}(t-1) & \text{otherwise.} \quad (13) \end{cases}$$

Once the updated values of SP and FC are computed we can compute the total values of each force at every discrete time step t according to the formula

$$SP_l(t) = \sum_k SP_{kl}(t) \tag{14}$$

and

$$FC_l(t) = \sum_k FC_{kl}(t). \tag{15}$$

These total values represent the aggregate staying power (SP) and the theoretical combat power (FC) at the end of the time step t.

4. Model Interpretation

The above model was implemented using a computer program that calculates attrition at each time step against the groups under attack. Specifically, the program does the following:

- Starts and stops the continuous fire based on the duration of the fire.
- Computes the attrition for both forces at each time step based on which groups of the opposing forces are the attackers and which are the targets of each attacker.
- Stops the engagement based on either the number of steps to be run (specified by the user), or the maximum acceptable percent loss in staying power of each force (specified also by the user), whichever is greater.

5. Discussion of the Model

The overall performance of the model can be summarized as follows [Ref. 8:p. 16]. The model

- Portrays naval forces as aggregations of the staying power and theoretical combat power of heterogeneous mixes of platforms.
- Models the engagement of these forces as a force-on-force attrition process with attrition suffered via continuous fire and/or through the impact of pulse weapons.
- Permits the user to vary the inputs concerning the time, strength, target, and duration of each force's fire in order to explore each force's tactical options.
- Computes attrition to the opposing forces simultaneously throughout the engagement and provides a result in terms of the percent SP and FC lost by each force.

We believe this model is a very credible and promising one. The main disadvantages that can be distinguished are:

- The model does not deal with missiles. We believe that the most important weapon in today's naval combat is the missile.
- To compute some values (SP for example), the model uses historical data. Thus we do not know if the computed values reflect today's technological developments. This weakness cannot be corrected because the existing data from recent naval combats are not sufficient.
- The model does not incorporate human factors.

C. COMPARISON OF THE TWO MODELS

Although the two models presented above provide very different approaches and are constructed for different kinds of combat, we now make a side-by-side comparison, being aware that it is a crude one.

- Both models use the term *theoretical combat power* with roughly the same meaning.
- Dupuy's *combat effectiveness* and Beall's *weapon effectiveness* are somewhat related, but they have different meanings. CEV, in Dupuy's model, refers explicitly to human factors and its values are not restricted. Weapon effectiveness (PC) is a probability (and is restricted between 0.0 and 1.0).
- Dupuy's *force strength* also is related to Beall's *weapon effectiveness* but the two terms refer to different quantities (weapon effectiveness is a probability whereas force strength varies from 500 to 500,000).
- Beall's *effective combat power* and Dupuy's *combat power* have roughly the same meaning.
- Beall's equation $EFC = FC \times PC$ and Dupuy's $P = S \times V_f \times Q$ are similar, but Beall makes no provision for human factors.
- The disadvantages of the QJM model are that it does not account for (1) tactics on the battlefield, (2) scouting-antiscouting effects, (3) staying power of platforms and pulse weapons, and (4) naval battle data.
- The main disadvantage of Beall's model is that it does not take into account human factors and the factor of chance or luck.

IV. A MODERN NAVAL COMBAT MODEL

A. INTRODUCTION

As previously emphasized, today's sea battles and sea control are based primarily on one weapon: the missile. For this reason we will develop a model to represent missile combat. As stated before, missiles are considered to be pulse weapons because they deliver instantaneously a great amount of firepower against a selected target. The model is developed as a force-on-force attrition process and will be based on a simple concept.

What is of most interest in a naval combat? Generally, the losses suffered on each side and, consequently, the remaining staying power and theoretical combat power of each force after losses have occurred. We previously defined staying power as the number of hits a platform can absorb before suffering a firepower kill. So, if we consider a force fighting as a single unit, we can define the aggregate staying power of a force as the sum of the total theoretical number of hits each platform can absorb and still continue fighting. If we consider the ratio of hits received by one force in one firepower pulse divided by its staying power, the result will be the percentage loss of this force incurred in this pulse (that is, hits received divided by hits that can be absorbed

before being destroyed). In the model this percentage loss will be computed in discrete time steps for each force and then used to determine the remaining staying power and theoretical combat power of each force at the end of every discrete time step.

We will also incorporate in our model the effect of human factors issues such as scouting effectiveness, training, morale, and leadership. These effects will be incorporated implicitly in terms of some degraders in combat effectiveness of each force.

In the model we deal with a particular type of missile, a typical surface-to-surface antiship missile. This is necessary because the specifications for different kinds of missiles vary enormously (technology used, explosive material, weight of the explosive material, speed, guidance technique, and so forth). Of course the model is general enough so that it can be adjusted easily to include other types of missiles by assigning appropriate values to the parameters of the model. For the simple case of missiles with approximately the same type of explosive material, technology, and guidance technique, we use a single multiplier, W_m, to account for the different weights of the explosive material. The W_m multiplier can be ignored when the opposing forces both are using the same nominal missile.

B. MODEL DEVELOPMENT

1. Individual Platform Case

In the context of the combat theory already presented, we redefine the main parameters used in our model.

Indices used in the model are as follows:

j = Platform of the Blue force

j'= Platform of the Red force

k = Group of platforms constituting the Blue force

k'= Group of platforms constituting the Red force

b = Blue force

r = Red force

Staying Power (SP): The number of hits a platform can absorb before suffering a firepower kill. Staying power depends on the kind of enemy missile. Although the data used to develop the formula for staying power in Beall's model (page 41) are drawn from World Wars I and II, the fact that there is little new data, especially for missiles, compels us to use this same formula for our approximation. Moreover, the formula has been tested for different values of full load displacement, and for an average or nominal type of missile the resulting damage was found to be reasonable. Thus, the staying power of platform j in group k for the Blue force is given by the equation

$$SP_{jkb} = 0.070 \times (\textit{full load displacement})^{1/3}. \qquad (1)$$

We note here that SP was measured by Beall in units of 1000-pound bomb equivalent (TPBE) hits. We assume that our nominal missile has a destructive value of one TPBE. This is done for simplicity, without deviating significantly from reality. Therefore, for the remainder of the thesis SP is in units of nominal missile hits, and is the number of nominal hits a ship can take before firepower kill, as a function of its displacement.

<u>Theoretical combat power</u> (P): This is the number of missiles that can be fired from a unit in a single salvo. The number of missiles that can be fired in a salvo by unit j in group k for the Blue force is given by the equation

$$P_{jkb} = M_{jkb} \times W_m \qquad (2)$$

where: M_{jkb} = The theoretical number of missiles that a unit j of group k in the Blue force can fire in a single salvo.

W_m = A multiplicative factor to be used for missiles based on approximately the same technology to account for the different weights of explosive material. For example, if

one side uses a missile with twice as much explosive material as our nominal missile, then W_m is 2.0 and the side has double the theoretical combat power. The multiplier W_m can be ignored if both sides use a missile that is equivalent of our nominal one-TPBE missile.

Effective combat power or combat effectiveness (E): The number of missiles that hit their target per salvo. The effective combat power of platform j' in group k' in the Red force is given by

$$E_{j'k'r} = M_{j'k'r} \times W_m \times PR_{j'k'r} \tag{3}$$

where: $PR_{j'k'r}$ = The probability that a missile fired from unit j' in group k' in the Red force hits its target.

The value of PR can be calculated as follows:

$$PR_{j'k'r} = = H - \left(H \times \frac{N_{jkb}}{M_{j'k'r}}\right) \tag{4}$$

where: H = For each missile the probability of striking an undefended target (for the same type of missiles H is the same for all units in the force). In other words H represents the firing accuracy and is given for each type of missile.

N_{jkb} = The number of missiles a defender (j platform in k group in the Blue force) can shoot down per salvo (the best he can do).

The formula given in Equation (4) was derived from the following reasoning. H is the conditional probability of a hit, given that the missile will not be shot down, or mathematically H = P[hit/not shot down]. Clearly P[hit/shot down] = 0. Since we cannot know the exact probability that the defender will shoot down a missile, we can use the estimate P_s. We know that an estimate of this probability is given by $P_s = N_{jkb}/M_{j'k'r}$. So using the law of total probability we have

$$PR = P(hit) = H\times(1-P_s) + 0\times P_s = H\times(1-P_s) = H\times(1-N_{jkb}/M_{j'k'r})$$

which is equivalent to Equation (4).

For instance, suppose that a Blue unit fires against a Red one. If the Blue unit can fire four missiles in a salvo, the Red unit defending can shoot down two missiles, and the firing accuracy (H) for the Blue missile is 0.8, then the probability that a Blue missile will hit the Red unit (PR) is 0.8 - (0.8 × 2/4), or 0.4.

Substituting Equation (4) in Equation (3), the effective combat power of platform j' in group k' of the Red force can be written as

$$E_{j'k'r} = (M_{j'k'r} \times W_m \times H) - (N_{jkb} \times W_m \times H) . \tag{5}$$

Although the above formula is mathematically correct, observe that the righthand part of the equation applies when the defender shoots down missiles without knowing if they are going to hit him. For a modern naval missile combat frequently the defender can determine which missiles are a threat and shoots down only the ones that will hit him. For this case H does not apply to the second term in the righthand part of the equation, resulting in the following modified equation:

$$E_{j'k'r} = (M_{j'k'r} \times W_m \times H) - (N_{jkb} \times W_m) . \tag{6}$$

Discussion

Which is preferable, Equation (5) or Equation (6)? Consider separately, first, offensive combat power $P_{j'k'r} = M_{j'k'r} \times W_m$ (interpreted as missiles that can be fired per salvo by platform j' in Red force), and second, defensive combat power $D_{jkb} = N_{jkb} \times W_m$ (interpreted as missiles that can be shot down per salvo by platform j in Blue force). The effective combat power of platform j', which represents the net number of missiles that could hit, is then given by $E_{j'k'r} = P_{j'k'r} - D_{jkb}$.

Case 1 (Equation (5))

Shooter fires $M_{j'k'r}$ missiles, and defender shoots down N_{jkb} of them (without knowing which will hit). In this case we have the following equation:

$$E_{j'k'r} = H \times (P_{j'k'r} - D_{jkb}) = W_m \times H \times (M_{j'k'r} - N_{jkb}).$$

So, we examine which missiles hit after defender acts. For instance, suppose a Red platform fires against Blue using nominal missiles (W_m is ignored). Furthermore, suppose the missile hit probability H = 0.5. Red fires a salvo of four missiles. Blue shoots down two without knowing which of the four will hit. The remaining two will hit Blue with probability 0.5, and so one may be expected to hit. Using the formula $E_{j'k'r} = H \times (M_{j'k'r} - N_{jkb}) = 0.5 \times (4-2) = 1$, we see that the result is consistent. This model is useful when the defender uses AAW surface-to-air missiles at long range before he knows which enemy missile will acquire and hit him.

Case 2 (Equation (6))

Shooter again fires $M_{j'k'r}$ missiles. The defender sees which of the missiles (that is, $H \times M_{j'k'r}$) will hit. Thus, the defender shoots down N_{jkb} out of $H \times M_{j'k'r}$ missiles. In this case we use Equation (6):

$$E_{j'k'r} = H \times P_{j'k'r} - D_{jkb} = W_m \times (H \times M_{j'k'r} - N_{jkb}).$$

Using the same scenario as in case 1, a Red platform fires a salvo of four missiles against Blue with hit probability H = 0.5. The expected number that can hit is therefore two. The defender, using the means he possesses, is informed which two will hit and shoots both of them down. Thus, he will suffer zero hits. Using our formula we find $E_{j'k'r} = H \times M_{j'k'r} - N_{jkb} = (0.5 \times 4) - 2 = 0$. We conclude that Equation (6) is consistent.

Case 2 is most appropriate for our situation, in which "point" (close in) defenses are used. This situation occurs frequently against sea-skimming missiles in the terminal defense phase. We think Equation (6) is closer to reality for a modern missile naval combat, and hereafter in our development assume Equation (6) to be the case. When we suspect the defender's capabilities are overestimated, the value of N_{jkb} (number of defeated missiles) should be adjusted appropriately.

Equation (6) represents the effective combat power of a single Red platform firing against a single defended Blue platform, and it is measured in hits inflicted on the Blue platform. It will be convenient for our model to define effective combat power of the attacking force in terms of the destroyed staying power of the defending force. In order to accomplish this we divide Equation (6) by the staying power (SP) of the defending (Blue) force. We call the resulting fraction of destroyed staying power LOSS. If Red is attacking and Blue defending, the fraction of destroyed staying power of platform j in group k of Blue, (which also represents the effective combat power of platform j' in group k' of Red) is as follows:

$$LOSS_{jkb} = \frac{(M_{j'k'r} \times W_m \times H) - (N_{jkb} \times W_m)}{SP_{jkb}}$$

$$= \frac{W_m}{SP_{jkb}} \times \left[(M_{j'k'r} \times H) - N_{jkb} \right]. \qquad (7)$$

Note that $LOSS_{jkb}$ should be a number between 0.0 and 1.0. A negative value of LOSS means that the Blue force is able to shoot down more missiles than the Red force is able to fire in a single salvo. In that case we set the Blue force $LOSS_{jkb}$ equal to 0.0. When $LOSS_{jkb}$ has a value greater than 1.0,

it means that the Red force fired more missiles than needed to completely destroy all of Blue's staying power, resulting in Red "overkill." In that case the Blue force has no remaining staying power and has suffered a firepower kill. Of course, if just the right number of missiles are not spread efficiently over the defenders, "overkill" is desirable.

Up to now, and before we introduce human factors, first observe that our model has an advantage over Beall's model: our model takes into account the capability of the defending force. Next we incorporate several human factor components.

In the combat theory presented in Chapter II, we emphasized the importance of scouting. Scouting is defined as the means to gather any type of useful information about the enemy. Define σ to be the scouting function with values between 0.0 and 1.0. The scouting function σ is applied to the attacking force. Also, the readiness of the defending force to defend is clearly important. Define τ to be the alertness modifier for the defending force, with values again between 0.0 and 1.0. Incorporating the scouting function σ and the alertness modifier τ, Equation (6) becomes

$$LOSS_{jkb} = \frac{W_m}{SP_{jkb}} \times (\sigma_r \times M_{fKr} \times H \; - \; \tau_b \times N_{jkb}) \qquad (8)$$

where: σ_r = Scouting function of the attacking Red force (same for all the red units)

τ_b = Alertness modifier of the defending Blue force (same for all the blue units).

If the attacking force is fully informed of its opponent's posture, and the defending force is fully alert, then $\sigma_r = \tau_b = 1.0$ and Equation (8) reduces to Equation (7). If the attacking force has no information about the enemy, then $\sigma_r = 0.0$ and there are no hits ($LOSS_{jkb}$ will be negative or 0.0). If the Blue force has no information about the enemy, they have no alertness. The Red force then ambushes Blue using effective scouting. In this case $\sigma_r = 1.0$ and $\tau_b = 0.0$, resulting in the following loss equation:

$$LOSS_{jkb} = \frac{W_m}{SP_{jkb}} \times (\sigma_r \times M_{fKr} \times H) \, . \qquad (9)$$

Analysis of this last equation helps us understand the importance of surprise in a naval combat.

Now, what procedure should we use for determining σ and τ for both sides? Our recommendation is that the user should assign a value between 0.0 and 1.0 to both the scouting and alertness functions, according to the specific scenario. If the model is used to evaluate historical battles, then the existing historical data will determine the appropriate values. If the model is used to estimate the outcome of a future battle, then the tactical commander will base his estimate on the specific tactical situation. If the tactical commander believes he is not able to assign values for σ and τ, then he also will not be able to assess the weapons effectiveness of his or the opposing side. As will be seen from the numerical examples that follow, scouting and alertness effectiveness are as decisive as firepower.

Up to now we have assumed full combat potential on both sides. However, as discussed in Chapter II, combat power is determined not by the designed combat potential, but by the available combat potential. We defined $M_{j'}$ as the number of missiles the unit j' can fire in a single salvo. However, this number is the theoretical maximum, or the "best" that unit j' can do. The real number of missiles that can be fired by a unit in action is a function of personnel training, morale, leadership, and sometimes chance or luck. In general, human factors come into play. In order to introduce these effects, we use a multiplicative degrader m that has a value between

0.0 and 1.0.[10] If the leader exploits every possible factor, personnel training and morale are at their maximum levels, and there is no accidental failure, then the particular unit can fire M missiles per salvo and m = 1.0. If, on the other hand, personnel morale is low (possibly after severe casualties have occurred) and it is impossible to return the enemy's fire, then m = 0.0. In all other cases, m has a value between 0.0 and 1.0. Hence Equation (7) can be modified as follows:

$$LOSS_{jkb} = \frac{W_m}{SP_{jkb}} \times (\sigma_r \times M_{j'K r} \times m_{j'K r} \times H \;-\; \tau_b \times N_{jkb}) \,. \quad (10)$$

Using a similar argument, it is reasonable to assume that generally the defender will not shoot down N missiles, but rather N×n, where n is a multiplicative factor with values between 0.0 and 1.0. In effect, n acts with N in exactly the same way that m acts with M. Incorporating this refinement, the percentage loss of staying power for the defending (Blue) platform j in group k (which also represents the effective combat power of platform j' fired against j for the Red force), has the representation

[10]Note that m affects both launch availability (M) and firing accuracy (H) caused by deficiencies in leadership, morale, and training.

$$LOSS_{jkb} = \frac{W_m}{SP_{jkb}} \times \left[(\sigma_r \times M_{j'k'r} \times m_{j'k'r} \times H) - (\tau_b \times N_{jkb} \times n_{jkb}) \right]. \quad (11)$$

Again, if the model is used to evaluate historical battles from existing data, it would not be difficult to determine the appropriate values for m and n for both sides. When a commander uses the model for estimating the outcome of a future naval combat, then he must make an estimate of the values of m and n based on the particular tactical scenario, his personnel's morale and training, and all the information he has about the enemy. An in-depth study of recent historical battles can greatly help to determine the proper values for these two multipliers.

As noted previously, the percentage loss of each force and the remaining staying power and theoretical combat power for both forces will be computed in discrete time steps. If we define $SP_{jkb}(t-1)$, and $P_{jkb}(t-1)$ to be the staying power and theoretical combat power respectively of platform j in group k for Blue at the end of the time step (t-1), then using $LOSS_{jkb}$ (Equation 11), we can compute the remaining staying power and theoretical combat power of platform j in group k for the Blue force at the end of time step t as follows:

$$SP_{jkb}(t) = \begin{cases} SP_{jkb}(t-1) \times (1-LOSS_{jkb}) & \forall j \text{ under attack} \\ SP_{jkb}(t-1) & \text{otherwise.} \end{cases} \tag{12}$$

$$P_{jkb}(t) = \begin{cases} P_{jkb}(t-1) \times (1-LOSS_{jkb}) & \forall j \text{ under attack} \\ P_{jkb}(t-1) & \text{otherwise.} \end{cases} \tag{13}$$

It is reasonable to assume that after a unit has been hit (received a pulse) its ability to shoot down missiles (N) will be reduced according to its loss of staying power. So, after a unit suffers a hit we update the value of N as follows:

$$N_{jkb}(t) = \begin{cases} N_{jkb}(t-1) \times (1-LOSS_{jkb}) & \forall j \text{ under attack} \\ N_{jkb}(t-1) & \text{otherwise.} \end{cases} \tag{14}$$

Before proceeding to the aggregation of units into groups it would be helpful to clarify the concepts and equations developed so far by giving a numerical example. We consider the simplest case of naval combat; the one between two single platforms.

2. Example 1: Individual Platform Case

Assume that both the Red and Blue forces consist of one FF having the characteristics presented in Table IV. We give one advantage to the Red FF of being able to fire four missiles while the Blue FF is able to fire only three. We assume there are no reloads and each force can fire only one pulse. We assume also that the two forces have the same scouting and alertness potential and use the same type of missiles which are fired almost simultaneously. All the other characteristics and human factors issues are assumed to be the same for both forces. Since the same type of missiles (nominal) is assumed for both forces, the multiplicative factor W_m is omitted.

TABLE IV. CHARACTERISTICS OF RED AND BLUE SHIPS

Factor	Blue FF	Red FF
Full load displacement	$D_b = 4000$	$D_r = 4000$
Staying power (computed)	$SP_b = 1.11$	$SP_r = 1.11$
Missiles per salvo	$M_b = 3$	$M_r = 4$
Multipl. degrader for M	$m_b = 0.8$	$m_r = 0.8$
Shots down per salvo	$N_b = 2$	$N_r = 2$
Multipl. degrader for N	$n_b = 0.8$	$n_r = 0.8$
Scouting function	$\sigma_b = 0.8$	$\sigma_r = 0.8$
Alertness modifier	$\tau_b = 0.8$	$\tau_r = 0.8$
Probability of hit	$H = 0.9$	

Theoretical combat power for both forces:

Red force: $P_r = M_r = 4$ hits per salvo

Blue force: $P_b = M_b = 3$ hits per salvo

Percentage loss for both forces:

Red force: $LOSS_r = (\sigma_b \times M_b \times m_b \times H - \tau_r \times N_r \times n_r) / SP_r =$

$(0.8 \times 3 \times 0.8 \times 0.9 - 0.8 \times 2 \times 0.8)/1.11 = 0.40$

(or, a loss of 40% in the Red force)

Blue force: $LOSS_b = (\sigma_r \times M_r \times m_r \times H - \tau_b \times N_b \times n_b) / SP_b =$

$(0.8 \times 4 \times 0.8 \times 0.9 - 0.8 \times 2 \times 0.8)/1.11 = 0.92$

(or, a loss of 92% in the Blue force)

Remaining staying power and theoretical combat power

Red force: $SP_r = SP_r \times (1 - LOSS_r) = 1.11 \times (1-0.40) = 0.67$

$P_r = P_r \times (1 - LOSS_r) = 4 \times (1-0.40) = 2.4$

Blue force: $SP_b = SP_b \times (1 - LOSS_b) = 1.11 \times (1-0.92) = 0.09$

$P_b = P_b \times (1 - LOSS_b) = 3 \times (1-0.92) = 0.24$

So, after a battle in which the two sides exchange missile salvoes, the Red FF has a 40% loss with remaining staying power 0.67 (instead of its original 1.11) and remaining theoretical combat power of 2.4 (instead of 4). The Blue FF has a loss of 92% with remaining staying power 0.09 (instead of 1.11) and theoretical combat power of 0.24 (instead of 3). However, neither side has reloads, so the missile battle is over. We observe that Blue has suffered almost a firepower kill, while Red has more than half of its initial staying power (SP) and theoretical combat power (P). This is noteworthy because Red started with only a 4:3 advantage in combat power (missiles fired) but no advantage in defensive firepower or in other respects.

Next let's consider the same scenario, except that the Blue FF due to effective scouting ($\sigma_b = \tau_b = 1.0$) has every possible knowledge about the enemy and is able to ambush the

better-armed Red FF. Red is not totally surprised, however, and has defensive alertness ($\tau_r = 0.8$). After the attack, Red FF gains targeting information about the enemy ($\sigma_r = 0.8$) and returns the fire, but only after being hit by the pulse of Blue missiles. All other values are kept exactly the same as before. In this situation we have the following computations:

<u>Percentage loss for Red force:</u>

$$LOSS_r = (\sigma_b \times M_b \times m_b \times H - \tau_r \times N_r \times n_r) \ / \ SP_r$$

$$= (1.0\times3\times0.8\times0.9 - 0.8\times2\times0.8)/1.11 = 0.79$$

(or, a loss of 79% in the Red force)

<u>Remaining staying power and theoretical combat power for Red force:</u>

$$SP_r = SP_r\times(1-LOSS_r) = 1.11\times(1-0.79) = 0.23$$

$$P_r = P_r\times(1-LOSS_r) = 4\times(1-0.79) = 0.84$$

So, the Red force can return the fire with only 0.84 combat power remaining instead of 4. The result of Red's reduced salvo is

<u>Percentage loss for Blue force:</u>

$$LOSS_b = (\sigma_r \times M_r \times m_r \times H - \tau_b \times N_b \times n_b) \ / \ SP_b$$

$$= (0.8\times0.84\times0.8\times0.9 - 1.0\times2\times0.8)/1.11 = -1.0$$

(or, a loss of 0.0% in the Blue force)

<u>Remaining staying power and theoretical combat power for Blue force:</u>

The negative $LOSS_b$ indicates that the Blue FF is able to shoot down more missiles than Red can now fire. Thus, Blue has zero losses, and 100% of its staying power and theoretical combat power remain, while Red suffers about 80% losses. This example illustrates the importance of surprise and effective scouting in a naval combat: a force with inferior combat potential can defeat a stronger force if it exploits its advantage in human factors. History justifies our reasoning.

3. Aggregation of Units into Groups

As described in the combat theory chapter, the term "group" refers to the subdivision of a force and a group may consist of several units. If we consider a group firing as a unit, then the aggregate staying power of group k in the Red force is given by

$$SP_{kb} = \sum_{j \in k} SP_{jkb} \qquad \forall\, k. \tag{15}$$

The aggregate percentage loss of group k of the (defending) Blue force, which also represents the destroyed staying power of group k, or equivalently the aggregate effective combat power of group k' of the (attacking) Red force (measured in destroyed staying power of the group under attack) is given by

$$LOSS_{jkb} = \frac{\sigma_r \times H \times W_m \times \sum_{f \text{ can be used}} M_{f'K'r} \times m_{f'K'r} - \tau_b \times W_m \times \sum_j N_{jkb} \times n_{jkb}}{SP_{kb}}. \qquad (16)$$

We note here that the first summation symbol in Equation (16) is used to sum the missiles from all the platforms belonging to group k' that fire missiles. In other words, for a particular situation at hand, there may be some ships that do not fire (because of the formation, or because there are friendly ships or land in the line of fire, or because their weapons fail). For this reason, missiles only from the ships that actually do fire a particular salvo are summed. Here then is another important domain where leadership plays a critical role: a good leader exploits every possible factor so that all ships are able to fire at the crucial moment.

If $SP_{kb}(t-1)$ is the staying power of group k in the Blue force at the end of time step (t-1), then the aggregate staying power of all Blue groups under attack is given by

$$SP_{kb} = \sum_{k \text{ being attacked}} SP_{kb}(t-1). \qquad (17)$$

Also, if $P_{k'r}(t-1)$ is the theoretical combat power of group k' belonging to the Red force at the end of time step (t-1), the aggregate theoretical combat power of the attacking (Red) groups is given by

$$P_{K'r} = \sum_{K' \text{ is firing}} P_{K'r}(t-1). \qquad (18)$$

Finally, if the aggregate percentage loss Equation (16) is applied to the staying power and theoretical combat power, their aggregate remaining values are computed iteratively from the previous time step according to the following equations:

$$SP_{kb}(t) = \begin{cases} SP_{kb}(t-1) \times (1-LOSS_{kb}) & \forall k \text{ under attack} \\ SP_{kb}(t-1) & \text{otherwise.} \end{cases} \qquad (19)$$

$$P_{kb}(t) = \begin{cases} P_{kb}(t-1) \times (1-LOSS_{kb}) & \forall k \text{ under attack} \\ P_{kb}(t-1) & \text{otherwise.} \end{cases} \qquad (20)$$

Also, the percentage loss can be applied to the ability (N) of the group to defend (shoot down missiles). At the end of each discrete time step we update the value of N as follows:

$$N_{kb}(t) = \begin{cases} N_{kb}(t-1) \times (1-LOSS_{kb}) & \forall k \text{ under attack} \\ N_{kb}(t-1) & \text{otherwise.} \end{cases} \tag{21}$$

When we have computed the updated values of SP and P for the attacking groups and the groups under attack for both forces, we use the following formulas to calculate the remaining total values for each force at every discrete time step t:

$$SP_b(t) = \sum_k SP_{kb}(t), \tag{22}$$

$$P_b(t) = \sum_k P_{kb}(t). \tag{23}$$

The values in Equations (22) and (23) represent the aggregates of the remaining staying power and theoretical

combat power at the end of the time step t. These values are then used to determine the naval combat outcome.

4. Input Parameters and Model Interpretation

In order to use the model, the user must determine the following inputs and model parameters:

- The composition of each force (number of groups, and number and type of units in each group).
- The staying power (SP) of each platform in both forces. The formula proposed in Equation (1) may be used to determine the SP.
- The number of missiles that can be fired per salvo (M) by each platform in both forces.
- The multiplicative factor W_m to account for the different weight in a missiles' warhead used by one force, if any. In order to avoid more complicated formulas, we assume homogeneity of missiles fired by each force in one pulse (in other words, W_m is the same for every unit in the force in each discrete time step t). It is easy to adjust the model to accommodate cases where different units in one force use different kinds of missiles in the same pulse, simply by summing all W_{mj} for every unit j in the force.
- The number of missiles (N) that theoretically can be shot down by each unit in both forces.
- The theoretical probability of hit (H) on an undefended target for the type of missile used in the model. We assume that both forces use the same type of missiles. If not, a different value of H can be used for each force.
- The scouting function σ, the alertness modifier τ, and the multiplicative degraders m and n to account for the effect of human factor issues for all platforms in both forces.
- Generally, the length of the discrete time steps. At the end of each step the remaining staying power and theoretical combat power for both forces will be calculated. Here is assumed that the duration of the discrete time steps is such that, during one time step, each force receives no more than one pulse.

- Optionally, the breaking point of each force; in other words, the lowest acceptable loss percentage of each force below which that particular force will be considered defeated and the battle terminated.

- The maximum number of salvoes each side can fire. This depends on the availability of missiles. For a missile combat, because of the limited number of missiles a platform can carry, no more than three or four pulses are generally expected by each group.

- The platforms which will be the targets of the other groups. Each group is considered as fighting as a unit, and we treat the aggregate values for the whole group. In order to do this, we assume that all the units in a group are being targeted equally. So, if a unit in a group is not a target, then another unit from the same group cannot be simultaneously the target of two different units of the opposing group (in other words, we assume no overkill). Based on this assumption, the user must determine which groups of one force are the targets of which groups of the other. Clearly, if the two opposing forces consist of one group each, there is nothing to be determined.

We now describe the logical sequencing of the model. The losses and the values of the remaining staying power and theoretical combat power are calculated for each group in both forces at the end of each discrete time step. During one time step each group is hit by one pulse or not at all (never more than one pulse). If a particular group is hit, then its percentage losses are calculated which determines remaining staying power and new offensive and defensive theoretical combat power. In the next time step we use these new calculated values. Then for the particular time step at hand, we sum over all the groups of each force. Thus at the end of each time step we find the total remaining values of staying power and theoretical combat power for each force. We

terminate the procedure when one of the forces reaches its break point, or after a specified number of time steps, whichever comes first. At the end, the user observes the losses on both sides as well as the remaining staying power and theoretical combat power of each force. Implicitly the side which won (or will win) is the side that eliminates the other's staying power, or forces the other to end the battle by the break point criterion. Another numerical example will clarify the process.

5. Example 2: Aggregated Force Case

Consider now a more complicated scenario than presented in Example 1. The Red force consists of four FFs and the Blue force of only three FFs. To keep the analysis as clear as possible, assume that all the FFs have the same specifications and carry the same type of missiles. The relevant values for each force are presented in Table V. Since the same type of missile is assumed for both forces, the multiplicative factor W_m can again be omitted.

TABLE V. CHARACTERISTICS OF RED AND BLUE SHIPS

Factor	Blue FF (3 ships)	Red FF (4 ships)
Full load displacement	$D_b = 4000$	$D_r = 4000$
Staying power for each ship (computed)	$SP_b = 1.11$	$SP_r = 1.11$
Missiles per salvo	$M_b = 3$	$M_r = 3$
Multipl. degrader for M	$m_b = 0.7$	$m_r = 0.7$
Shots down per salvo	$N_b = 2$	$N_r = 2$
Multipl. degrader for N	$n_b = 0.8$	$n_r = 0.8$
Scouting function	$\sigma_b = 0.8$	$\sigma_r = 0.8$
Alertness modifier	$\tau_b = 0.8$	$\tau_r = 0.8$
Probability of hit	$H = 0.8$	

Note that we are assuming the two sides have the same scouting function and the same multiplicative degraders m and n. Hence, at time t both forces are firing a salvo. We assume further that the battle terminates after three pulses from each side, or when a side reaches 80% loss, whichever comes first.

Initial staying power for both forces (aggregated):

$$Red\ force\text{:}\quad SP_r = 4 \times 1.11 = 4.44$$

$$Blue\ force\text{:}\quad SP_b = 3 \times 1.11 = 3.33$$

Note that remaining theoretical combat power P is expressed for one ship. At the end, if we need the total remaining theoretical combat power of the whole force we multiply it by the number of ships in the force.

- At the end of the first discrete time step we have

Blue force

Percentage loss of the total Blue force:

$LOSS_b = (\sigma_r \times 4 \times M_r \times m_r \times H - \sigma_b \times 3 \times N_b \times n_b) / SP_b$

$= (0.8 \times 4 \times 3 \times 0.7 \times 0.8 - 0.8 \times 3 \times 2 \times 0.8)/3.33 = 0.46$

(or, a loss of 46% in the Blue force)

Remaining staying power:

$SP_b = SP_b \times (1 - LOSS_b) = 3.33 \times (1 - 0.46) = 1.8$ (for entire force)

Updated P and N:

$P_b = M_b \times (1 - LOSS_b) = 3 \times (1 - 0.46) = 1.62$ (for each ship)

$N_b = N_b \times (1 - LOSS_b) = 2 \times (1 - 0.46) = 1.08$ (for each ship)

Red force

Percentage loss:

$LOSS_r = (\sigma_b \times 3 \times M_b \times m_b \times H - \sigma_r \times 4 \times N_r \times n_r) \ / \ SP_r$

$= (0.8 \times 3 \times 3 \times 0.7 \times 0.8 - 0.8 \times 4 \times 2 \times 0.8)/4.44 = -0.24$

(the negative value means no loss in the Red force)

Remaining staying power and theoretical combat power:

$SP_r = 4.44$ (same as before)

$P_r = 4$ (same as before)

$N_r = 2$ (same as before)

- At the end of the second time step (assuming that both forces fired the second pulse simultaneously) we will have the following values:

Blue force

Percentage loss:

$LOSS_b = (\sigma_r \times 4 \times M_r \times m_r \times H - \sigma_b \times 3 \times N_b \times n_b) \ / \ SP_b$

$= (0.8 \times 4 \times 3 \times 0.7 \times 0.8 - 0.8 \times 3 \times 1.08 \times 0.8)/1.8 = 1.8$

(or, a loss of 100% in the Blue force)

So, at the end of the second time step the entire Blue force has suffered a firepower kill while the Red force has essentially no damage. This should be evident without calculations. The Red force suffered no damage at all after the first pulse delivered by an undamaged Blue force. In the

second pulse the Blue force has now suffered a loss of 45% and the weaker force will still be unable to damage the Red force. The value of $LOSS_b$ (greater than 1.0) after the second pulse declares that Blue FF's received more hits (overkill) than necessary to put them all out of action.

Let's retain the same basic scenario but alter the human factor values slightly. The new values for both forces are shown underlined in Table VI.

TABLE VI. CHARACTERISTICS OF THE RED AND BLUE SHIPS

Factor	Blue FF (3 ships)	Red FF (4 ships)
Full load displacement	$D_b = 4000$	$D_r = 4000$
Staying power for each ship (computed)	$SP_b = 1.11$	$SP_r = 1.11$
Missiles per salvo	$M_b = 3$	$M_r = 3$
Multipl. degrader for M	$m_b = 0.7$	$m_r = 0.6$
Shots down per salvo	$N_b = 2$	$N_r = 2$
Multipl. degrader for N	$n_b = 0.8$	$n_r = 0.6$
Scouting function	$\sigma_b = 0.85$	$\sigma_r = 0.75$
Alertness modifier	$\tau_b = 0.85$	$\tau_r = 0.75$
Probability of hit	$H = 0.8$	

In summary, the Red force now has less information about the enemy and less defensive ability (0.75 versus 0.85). Moreover $m_r = 0.6$ and $n_r = 0.6$ (not well trained, low morale, and so forth). We assume also that the Red force is able to return fire before receiving its opponent's pulse. In this case we again have an exchange of fire.

- At the end of the first discrete time step we have

<u>Blue force</u>

<u>Percentage loss of the total Blue force:</u>

$$LOSS_b = (\sigma_r \times 4 \times M_r \times m_r \times H - \tau_b \times 3 \times N_b \times n_b) \;/\; SP_b$$

$$= (0.75\times4\times3\times0.6\times0.8 - 0.85\times3\times2\times0.8)/3.33 = 0.07$$

(or, a loss of 7% of the Blue force)

<u>Remaining staying power:</u>

$SP_b = SP_b \times (1-LOSS_b) = 3.33 \times (1-0.07) = 3.1$ (for entire force)

<u>Update P and N:</u>

$P_b = M_b \times (1-LOSS_b) = 3 \times (1-0.07) = 2.79$ (for each ship)

$N_b = N_b \times (1-LOSS_b) = 2 \times (1-0.07) = 1.86$ (for each ship)

<u>Red force</u>

<u>Percentage loss for the total Red force:</u>

$$LOSS_r = (\sigma_b \times 3 \times M_b \times m_b \times H - \tau_r \times 4 \times N_r \times n_r) \;/\; SP_r$$

$$= (0.85\times3\times3\times0.7\times0.8 - 0.75\times4\times2\times0.6)/4.44 = 0.154$$

(or, a loss of 15.4% of the Red force)

Remaining staying power:

$SP_r = SP_r \times (1-LOSS_r) = 4.44 \times (1-0.154) = 3.75$ (for entire force)

Update P and N:

$P_r = M_r \times (1-LOSS_r) = 3 \times (1-0.154) = 2.54$ (for each ship)

$N_r = N_r \times (1-LOSS_r) = 2 \times (1-0.154) = 1.69$ (for each ship)

- At the end of the second discrete time step we have

Blue force

Percentage loss:

$$LOSS_b = (\sigma_r \times 4 \times M_r \times m_r \times H - \tau_b \times 3 \times N_b \times n_b) \;/\; SP_b$$

$$= (0.75 \times 4 \times 2.54 \times 0.6 \times 0.8 - 0.85 \times 3 \times 1.86 \times 0.8)/3.1 = -0.04$$

(or, no loss at all in the Blue force)

Remaining staying power:

$SP_b = 3.1$ (same as before)

Update P and N:

$P_b = 2.79$ (same as before)

$N_b = 1.86$ (same as before)

Red force

Percentage loss:

$$LOSS_r = (\sigma_b \times 3 \times M_b \times m_b \times H - \tau_r \times 4 \times N_r \times n_r) \;/\; SP_r$$

$$= (0.85 \times 3 \times 2.79 \times 0.7 \times 0.8 - 0.75 \times 4 \times 1.69 \times 0.6)/3.75 = 0.25$$

(or, a loss of 25% of the remaining strength in the Red force)

Remaining staying power:

$SP_r = SP_r \times (1-LOSS_r) = 3.75 \times (1-0.25) = 2.8$ (for entire force)

Update P and N:

$P_r = M_r \times (1-LOSS_r) = 2.54 \times (1-0.25) = 1.9$ (for each ship)

$N_r = N_r \times (1-LOSS_r) = 1.69 \times (1-0.25) = 1.27$ (for each ship)

- At the end of the third discrete time step we have

Blue force

No losses, same values of SP_b, P_b, and N_b as before

Red force

Percentage loss:

$LOSS_r = (\sigma_b \times 3 \times M_b \times m_b \times H - \tau_r \times 4 \times N_r \times n_r) / SP_r =$

$(0.85 \times 3 \times 2.79 \times 0.7 \times 0.8 - 0.75 \times 4 \times 1.27 \times 0.6)/2.8 = 0.6$

(or, a loss of 60% of the remaining strength in the Red force)

Remaining staying power:

$SP_r = SP_r \times (1-LOSS_r) = 2.8 \times (1-0.6) = 1.12$ (for the entire force)

Update P and N:

$P_r = M_r \times (1-LOSS_r) = 1.9 \times (1-0.6) = 0.76$ (for each ship)

$N_r = N_r \times (1-LOSS_r) = 1.27 \times (1-0.6) = 0.51$ (for each ship)

Starting total theoretical combat power = $4 \times 4 = 12$

Remaining total theoretical combat power = $4 \times 0.76 = 3.04$

As you can see, after the third pulse the Blue force has a total loss of only 7% while the Red force has a loss of about 75% (only one fourth remains of its initial values of staying power and theoretical combat power). Now the importance of human factors is entirely evident: without consideration of human factors the Red force wins: but, according to our model, after incorporating human factors, the Blue force wins.

6. Computer Model

The model was implemented with a computer program for the given assumptions (see Appendix A). The program was tested for all the cases presented in Examples 1 and 2 and verified to work correctly. The outputs with the same results as are hand-calculated in the examples above are also given in Appendix A.

V. CONCLUSIONS AND RECOMMENDATIONS

In summary, the model developed in Chapter IV deals with naval surface missile battles. It exploits the fact that these battles have a particular characteristic: namely, that attrition in both forces is instantaneous and is incurred through the application of pulses or "salvos" from the opponents' forces. So the values of staying power and combat power, as well as the loss in each force, can be calculated at the end of every discrete time step or salvo. Based on salvo results at each step, at the end of the last time step one can assess the outcome of the particular battle.

We believe the model is a promising and appealing one. First, it seems to have an advantage over the QJM approach because it accounts for the dynamics of a battle. Second, it implements Beall's model and extends it in the following ways:

- It deals with missiles, the most effective weapon of today's naval battles.
- It takes into account the defensive ability of both forces.
- It integrates the scouting effectiveness and the alertness in defense for both opponents.
- It incorporates several important human factors that affect the outcome of a battle.

A possible weakness of the model is the provisional way human factors are quantified. How is it possible to assign values to these so called intangible variables? Our answer to that question is twofold. First, we did not try to assign particular values to the abstract human factors: willpower, morale, leadership, etc. Instead we tried to show the effect these human factors have upon the offensive and defensive capabilities of both opponents. Second, although everybody admits the great importance of human factors in a battle, usually no one is willing to indicate in what way they exert this influence. We have made a step into that difficult area and have given some analytical structure that will serve as a guide in the gathering of data in a most useful form to help future researchers.

What is needed next in order for this model to become a more useful tool? Our recommendations for future research are the following:

- Perform an extensive sensitivity analysis on the model. In other words, by changing slightly the different parameters, how does the model respond? Thus, it will be possible to check the reasonableness of the parameters and the assigned ranges to them.

- Analyze the small number of existing historical missile naval battles using the model. By doing this, the validity of the model can be assessed and (based on the existing historical data) a better sense of appropriate values for some of the model parameters (such as σ, τ, m, and n) can be obtained.

- Expand the program described in Appendix A to cover all possible cases, and then use the program to study existing historical data.

APPENDIX A

A. PROGRAM LISTING

The following program codes the naval combat model described in Chapter IV. The code is Fortran 77 and it was implemented on IBM 3033 AP mainframe computer in Naval Postgraduate School.

```
      PROGRAM NAVCOM

* ASSUMPTIONS
****************
*  1. Same type of missiles for both forces (the average missile)
*  2. Each force is consisting of one group
*  3. In the duration of each discrete time step we assume that both
*     forces receive one pulse, either both forces fire simultaneously
*     or the one force returns fire, after it has already received its
*     opponent's pulse (with reduced capabilities).
*
*
      INTEGER NB,NR,DB,DR,BUNITS,RUNITS,NPULSE,W,I,Z
      REAL LOSSB,LOSSR,SPB,SPR,SFB,SFR,MDB,MDR,NDB,NDR,UPDNB,UPDNR
      REAL TOTSPB,TOTSPR,REMSPB,REMSPR,REMPB,REMPR,H,BRPNTB,BRPNTR
      REAL MR,MB,TLOSSB,TLOSSR,AFB,AFR
```

```
* INITIALIZATION
      LOSSB = 0.0
      LOSSR = 0.0
      PRINT *, 'PLEASE, ENTER THE FOLLOWING DATA FOR BOTH OPPONENTS'
      PRINT *, 'BE CAREFUL, THE FIRST VALUE YOU ENTER TO BE FOR THE'
      PRINT *, 'BLUE FORCE AND THE SECOND FOR THE RED'
      PRINT *, 'NUMBER OF UNITS IN EACH FORCE (INTEGER)'
      READ *, BUNITS,RUNITS
      PRINT *, 'FULL LOAD DISPLACEMENT FOR BOTH FORCES (INTEGER)'
      READ *, DB,DR
      PRINT *, 'SCOUTING FUNCTION FOR BOTH FORCES (REAL)'
      READ *, SFB,SFR
      PRINT *, 'ALERTNESS FUNCTION FOR BOTH FORCES (REAL)'
      READ *, AFB,AFR
      PRINT *, 'NUMBER OF MISSILES A UNIT CAN FIRE PER SALVO FOR BOTH'
      PRINT *, 'FORCES (REAL)'
      READ *, MB,MR
      PRINT *, 'MULTIPLICATIVE DEGRADER FOR M FOR BOTH FORCES (REAL)'
      READ *, MDB,MDR
      PRINT *, 'NUMBER OF MISSILES A UNIT CAN SHOT DOWN IN ONE SALVO'
      PRINT *, 'FOR BOTH FORCES (INTEGER)'
      READ *, NB,NR
      PRINT *, 'MULTIPLICATIVE DEGRADER FOR N, FOR BOTH FORCES (REAL)'
      READ *, NDB,NDR
      PRINT *, 'THE BREAK POINT FOR BOTH FORCES (REAL BETWEEN 0 AND 1)'
```

```
PRINT *, 'NOTE: THIS IS THE PERCENTAGE OF THE INITIAL STAYING'
PRINT *, 'POWER BELOW WHICH THE BATTLE IS CONSIDERED TERMINATED'
PRINT *, 'IF YOU DO NOT WISH TO ASSIGN VALUES FOR BREAK POINT'
PRINT *, 'ENTER 0.0,0.0'
READ *, BRPNTB,BRPNTR
PRINT *, 'PROBABILITY OF HIT VERSUS UNDEFENDED TARGET (REAL)'
READ *, H
PRINT *, 'NUMBER OF DISCRETE TIME STEPS FOR THE PROGRAM TO BE'
PRINT *, 'EXECUTED'
READ *, NPULSE
UPDNB = NB
UPDNR = NR
TLOSSB = 1.0
TLOSSR = 1.0
SPB = 0.070*((REAL(DB))**(1.0/3.0))
SPR = 0.070*((REAL(DR))**(1.0/3.0))
TOTSPB = SPB*REAL(BUNITS)
TOTSPR = SPR*REAL(RUNITS)
REMSPB = TOTSPB
REMSPR = TOTSPR
REMPB = MB
REMPR = MR
I = 0
PRINT *, 'ARE BOTH FORCES FIRING SIMULTANEOUSLY THE PULSES?'
PRINT *, '(1 IF YES, 0 IF NO)'
```

```
      READ *, W
      IF (W .EQ. 0) GO TO 15
      PRINT *, ' '

* PROGRAM EXECUTION
*   Both forces fire simultaneously
*
 10   I =I + 1
         LOSSB=(SFR*REAL(RUNITS)*REAL(MR)*MDR*H-AFB*REAL(BUNITS)*UPDNB*
     +   NDB)/REMSPB
         IF (LOSSB .LT. 0.0) LOSSB = 0.0
         IF (LOSSB .GE. 1.0) LOSSB = 0.999
         REMSPB = REMSPB*(1.0-LOSSB)
         REMPB = REMPB*(1.0-LOSSB)
         UPDNB = UPDNB*(1.0-LOSSB)
         TLOSSB = 1-(REMSPB/TOTSPB)
*
         LOSSR=(SFB*REAL(BUNITS)*REAL(MB)*MDB*H-AFR*REAL(RUNITS)*UPDNR*
     +   NDR)/REMSPR
         IF (LOSSR .LT. 0.0) LOSSR = 0.0
         IF (LOSSR .GE. 1.0) LOSSR = 0.999
         REMSPR = REMSPR*(1.0-LOSSR)
         REMPR = REMPR*(1.0-LOSSR)
         UPDNR = UPDNR*(1.0-LOSSR)
         TLOSSR = 1-(REMSPR/TOTSPR)
```

```
*
          IF (((1-TLOSSB) .LE. BRPNTB) .AND.
     +((1-TLOSSR) .LE. BRPNTR)) GO TO 991
          IF ((1-TLOSSB) .LE. BRPNTB) GO TO 992
          IF ((1-TLOSSR) .LE. BRPNTR) GO TO 993
          MR = REMPR
          MB = REMPB
       IF (I .LT. NPULSE) GO TO 10
 15    CONTINUE
       IF (W .EQ. 0) THEN
          PRINT *, 'WHICH FORCE FIRES FIRST? NOTE: THAT MEANS THAT THE'
          PRINT *, 'OTHER FORCE RECEIVES THE PULSE FIRST AND THEN'

          PRINT *, 'RETURNS THE FIRE (0 FOR RED FORCE 1 FOR BLUE FORCE)'
          READ *, Z
          IF (Z .EQ. 1) GO TO 30
*
*  Red force fires first
*
 20       I = I + 1

               LOSSB=(SFR*REAL(RUNITS)*REMPR*MDR*H-AFB*REAL(BUNITS)*
     +         UPDNB*NDB)/REMSPB
               IF (LOSSB .LT. 0.0) LOSSB = 0.0
```

```
          IF (LOSSB .GE. 1.0) LOSSB = 0.999
          REMSPB = REMSPB*(1.0-LOSSB)
          REMPB = REMPB*(1.0-LOSSB)
          UPDNB = UPDNB*(1.0-LOSSB)
          TLOSSB = 1-(REMSPB/TOTSPB)
          IF ((1-TLOSSB) .LE. BRPNTB) GO TO 992
*
          LOSSR=(SFB*REAL(BUNITS)*REMPB*MDB*H-AFR*REAL(RUNITS)*
     +    UPDNR*NDR)/REMSPR
          IF (LOSSR .LT. 0.0) LOSSR = 0.0
          IF (LOSSR .GE. 1.0) LOSSR = 0.999
          REMSPR = REMSPR*(1.0-LOSSR)
          REMPR = REMPR*(1.0-LOSSR)
          UPDNR = UPDNR*(1.0-LOSSR)
          TLOSSR = 1-(REMSPR/TOTSPR)
          IF ((1-TLOSSR) .LE. BRPNTR*TOTSPR) GO TO 993
      IF (I .LT. NPULSE) GO TO 20
 30   CONTINUE
      IF (Z .EQ. 1) THEN
 40       I = I + 1
*
*  Blue force fires first
*
              LOSSR=(SFB*REAL(BUNITS)*REMPB*MDB*H-AFR*
     +        REAL(RUNITS)*UPDNR*NDR)/REMSPR
```

```
                IF (LOSSR .LT. 0.0) LOSSR = 0.0
                IF (LOSSR .GE. 1.0) LOSSR = 0.999
                REMSPR = REMSPR*(1.0-LOSSR)
                REMPR = REMPR*(1.0-LOSSR)
                UPDNR = UPDNR*(1.0-LOSSR)
                TLOSSR = 1-(REMSPR/TOTSPR)
                IF ((1-TLOSSR) .LE. BRPNTR*TOTSPR) GO TO 993
*
                LOSSB=(SFR*REAL(RUNITS)*REMPR*MDR*H-AFB*
     +          REAL(BUNITS)*UPDNB*NDB)/REMSPB
                IF (LOSSB .LT. 0.0) LOSSB = 0.0
                IF (LOSSB .GE. 1.0) LOSSB = 0.999
                REMSPB = REMSPB*(1.0-LOSSB)
                REMPB = REMPB*(1.0-LOSSB)
                UPDNB = UPDNB*(1.0-LOSSB)
                TLOSSB = 1-(REMSPB/TOTSPB)
                IF ((1-TLOSSB) .LE. BRPNTB*TOTSPB) GO TO 992
            IF (I .LT. NPULSE) GO TO 40
        ENDIF
      ENDIF
*
      CALL EXCMS('FILEDEF 10 DISK NAVCOM OUTPUT A')
*
      WRITE(10,94) 'AFTER',I,' PULSES THE RED FORCE HAS TOTAL LOSS',
     +      TLOSSR,'WITH REMAINING '
```

```
      WRITE(10,95) 'STAYING POWER',REMSPR,
     +              'AND REMAINING THEORETICAL COMBAT POWER ',REMPR
      WRITE(10,94) 'AFTER',I,'PULSES THE BLUE FORCE HAS TOTAL LOSS',
     +              TLOSSB,'WITH REMAINING'
      WRITE(10,95) 'STAYING POWER',REMSPB,
     +              'AND REMAINING THEORETICAL COMBAT POWER ',REMPB
      WRITE(10,96) 'THE PROGRAM IS TERMINATED HERE'
 94   FORMAT (3X,A,1X,I2,1X,A,1X,F5.2,1X,A)
 95   FORMAT (3X,A,1X,F5.2,1X,A,1X,F5.2)
 96   FORMAT (3X,A)
      STOP
991   CALL EXCMS('FILEDEF 10 DISK NAVCOM OUTPUT A')
      WRITE(10,996) 'BOTH FORCES REACHED THEIR BREAK POINT'
      WRITE(10,994) 'AFTER',I,'PULSES THE RED FORCE HAS TOTAL LOSS',
     +              TLOSSR,'WITH REMAINING'
      WRITE(10,995) 'STAYING POWER',REMSPR,
     +              'AND REMAINING THEORETICAL COMBAT POWER ',REMPR
      WRITE(10,994) 'AFTER',I,'PULSES THE BLUE FORCE HAS TOTAL LOSS',
     +              TLOSSB,'WITH REMAINING'
      WRITE(10,995) 'STAYING POWER',REMSPB,
     +              'AND REMAINING THEORETICAL COMBAT POWER ',REMPB
      WRITE(10,996) 'THE PROGRAM IS TERMINATED HERE'
994   FORMAT (3X,A,1X,I2,1X,A,1X,F5.2,1X,A)
995   FORMAT (3X,A,1X,F5.2,1X,A,1X,F5.2)
996   FORMAT (3X,A)
```

```
      STOP
992   CALL EXCMS('FILEDEF 10 DISK NAVCOM OUTPUT A')
      WRITE(10,999)'BLUE FORCE REACHED ITS BREAK POINT. RED FORCE WINS'
      WRITE(10,997) 'AFTER',I,' PULSES THE RED FORCE HAS TOTAL LOSS',
     +               TLOSSR,'WITH REMAINING'
      WRITE(10,998) 'STAYING POWER',REMSPR,
     +               'AND REMAINING THEORETICAL COMBAT POWER ',REMPR
      WRITE(10,997) 'AFTER',I,'PULSES THE BLUE FORCE HAS TOTAL LOSS',
     +               TLOSSB,'WITH REMAINING'
      WRITE(10,998) 'STAYING POWER',REMSPB,
     +               'AND REMAINING THEORETICAL COMBAT POWER ',REMPB
      WRITE(10,999) 'THE PROGRAM IS TERMINATED HERE'
997   FORMAT (3X,A,1X,I2,1X,A,1X,F5.2,1X,A)
998   FORMAT (3X,A,1X,F5.2,1X,A,1X,F5.2)
999   FORMAT (3X,A)
      STOP
993   CALL EXCMS('FILEDEF 10 DISK NAVCOM OUTPUT A')
      WRITE(10,899)'RED FORCE REACHED ITS BREAK POINT. BLUE FORCE WINS'
      WRITE(10,897) 'AFTER',I,' PULSES THE RED FORCE HAS TOTAL LOSS',
     +               TLOSSR,'WITH REMAINING'
      WRITE(10,898) 'STAYING POWER',REMSPR,
     +               'AND REMAINING THEORETICAL COMBAT POWER ',REMPR
      WRITE(10,897) 'AFTER',I,' PULSES THE BLUE FORCE HAS TOTAL LOSS',
     +               TLOSSB,'WITH REMAINING'
      WRITE(10,898) 'STAYING POWER',REMSPB,
```

```
+                'AND REMAINING THEORETICAL COMBAT POWER ',REMPB
      WRITE(10,899) 'THE PROGRAM IS TERMINATED HERE'
897   FORMAT (3X,A,1X,I2,1X,A,1X,F5.2,1X,A)
898   FORMAT (3X,A,1X,F5.2,1X,A,1X,F5.2)
899   FORMAT (3X,A)
      STOP
      END
```

B. SAMPLE OUTPUTS

We give below all the outputs generated by the above program for the examples presented in the main body of the thesis (Chapter IV).

Example 1 - Case A

```
AFTER  1  PULSES THE RED FORCE HAS TOTAL LOSS  0.40 WITH REMAINING
STAYING POWER  0.67 AND REMAINING THEORETICAL COMBAT POWER   2.40
AFTER  1 PULSES THE BLUE FORCE HAS TOTAL LOSS  0.92 WITH REMAINING
STAYING POWER  0.09 AND REMAINING THEORETICAL COMBAT POWER   0.25
THE PROGRAM IS TERMINATED HERE
```

Example 1 - Case B

```
AFTER  1  PULSES THE RED FORCE HAS TOTAL LOSS  0.79 WITH REMAINING
STAYING POWER  0.24 AND REMAINING THEORETICAL COMBAT POWER   0.85
AFTER  1 PULSES THE BLUE FORCE HAS TOTAL LOSS  0.00 WITH REMAINING
STAYING POWER  1.12 AND REMAINING THEORETICAL COMBAT POWER   3.00
THE PROGRAM IS TERMINATED HERE
```

Example 2 - Case A

```
BLUE FORCE REACHED ITS BREAK POINT. RED FORCE WINS
AFTER  2  PULSES THE RED FORCE HAS TOTAL LOSS  0.00 WITH REMAINING
```

STAYING POWER 4.47 AND REMAINING THEORETICAL COMBAT POWER 3.00

AFTER 2 PULSES THE BLUE FORCE HAS TOTAL LOSS 1.00 WITH REMAINING

STAYING POWER 0.00 AND REMAINING THEORETICAL COMBAT POWER 0.00

THE PROGRAM IS TERMINATED HERE

Example 2 - Case B

AFTER 3 PULSES THE RED FORCE HAS TOTAL LOSS 0.74 WITH REMAINING

STAYING POWER 1.18 AND REMAINING THEORETICAL COMBAT POWER 0.79

AFTER 3 PULSES THE BLUE FORCE HAS TOTAL LOSS 0.07 WITH REMAINING

STAYING POWER 3.11 AND REMAINING THEORETICAL COMBAT POWER 2.79

THE PROGRAM IS TERMINATED HERE

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